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Estimating and Forecasting Volatility of the Malaysian Stock Market Using a Combination of Kalman Filter and GARCH Models

Cheah Lee Hen*
Universiti Sains Malaysia

Zainudin Arsad**
Universiti Sains Malaysia

Abstract: The Kuala Lumpur Composite Index plays an important role as an indicator to the growth of investment in share equity and economic development in Malaysia. It has been an area of interest to investigate how changes in volatility affect share prices. The Kalman filter is an algorithm for sequentially updating a linear projection for the system. Since its development by Kalman and Bucy in the 1960s, the applications of the Kalman filter technique in financial time series have been very few and far in between. This study aims to investigate the nature and behaviour of the volatility of the Kuala Lumpur Composite Index return using a combination of Kalman filter and ARCH-type models. Three ARCH-type models (GARCH, GARCH-in-Mean and Exponential-GARCH) were considered. Monthly data from May 1986 to February 2005 were used. In general, the results strongly indicate the unsuitability of the constant variance Kalman filter model, suggesting the importance for modeling time varying volatility in the return series. When ARCH structure was taken into account in the conditional variance of the Kalman filter framework, the model was found to provide better results over the pure Kalman filter model. Further analysis also revealed that a combination of Kalman filter and ARCH-type models better fitted the KLCI series than the simple ARCH-type models. In conclusion, the Kalman filter model with time-varying conditional variance can better capture the behaviour of return series and successfully model the changing of variances.

Keywords: Kalman filter, GARCH, leverage effect, risk-return trade-off

1. Introduction

Bursa Malaysia, formerly known as Kuala Lumpur Stock Exchange, was established in 1974. It is under the purview of the Ministry of Finance and a Securities Commission. Bursa Malaysia computes an index for each of the main sectors traded on the bourse but the most widely followed by far is the Kuala Lumpur Composite Index (KLCI). It was introduced in 1986 and regarded as a barometer of the performance of the Malaysian stock market and the economy. The companies that make up the KLCI are some of Malaysia's largest public corporations and are among the most heavily traded issues in Bursa Malaysia. The number of component companies is 100 although the actual number may change from time to time. The KLCI is presently calculated and disseminated on a minute-by-minute basis.

*School of Mathematical Sciences, Universiti Sains Malaysia, 11800 Penang, Malaysia.
Email:*cheah_00@yahoo.com.my*
**School of Mathematical Sciences, Universiti Sains Malaysia, 11800 Penang, Malaysia.
Email: *zainudin@cs.usm.my*

Investors, regulators and brokers have all expressed concern over the level of stock market volatility. Information and knowledge of the behaviour of return volatility is important due to the fact that changes in volatility would affect share prices. For example, increases in volatility will produce a negative effect on share prices. The stock market crash in October 1987 and the subsequent drop in stock prices in 1989 left many people wondering whether stock prices have not become too volatile. Fama (1965) concluded that the movements of stock prices are best characterised by a random walk process. It will be a futile effort to investigate stock volatility if all information has been incorporated into prices. In the last couple of decades, the temporal relation between stock index and other macroeconomic variables has been of interest to academicians, regulators and practitioners. However, the sensitiveness of the relationships varies depending on the period of economic crisis or structural change.

The state-space model was used in this study to investigate the volatility of the Malaysian stock prices. The model was combined with the family of the ARCH model (ARCH, GARCH, GARCH-M and EGARCH, respectively). Comparisons among models were made and the best model was chosen to run the data before and after the economic crisis of 1997.

2. Literature Review

Arsad (2002) made use of the Kalman filter technique to analyse state-space model extensions of the Wilkie stochastic asset model. A model for the United Kingdom Retail Price Index (RPI) was proposed and investigated. The model allowed the local mean reversion level of inflation as well as inflation itself to be modelled stochastically. Results between the simpler Wilkie AR(1) and the proposed models were compared. The Kalman filter technique was also applied to a combined model for rate of inflation, equity dividend yields and dividend growth in order to investigate the effect of the equity dividend yields and dividend growth on the behaviour of futures rate of inflation. Finally, a possible simpler Kalman filter model was investigated by reducing the number of non-zero parameters in the model to avoid losing much of the explanatory power of the parameters.

Morris and Pfeffermann (1984) applied the Kalman filter technique to forecast festivals which did not fall on the same date in the Gregorian calendar from year to year. It was applied to three different series including Taiwan traffic volume, Israel work seekers and Israel tourist arrivals. For the first series, it was assumed that New Year affects a ten-day period, starting three days before the festival and that the extra traffic is distributed evenly throughout this period. The results showed a considerable increase in traffic volume around the date of the Chinese New Year from 21st January to 17th February. In addition, both Passover and the Jewish New Year were believed to significantly affect the second and third series respectively. It was also assumed that the Passover affects a twenty-three day period starting seventeen days before the festival and that the New Year affects a thirty-day period, starting five days before the festival.

Kato *et al.* (1995) investigated the relationship between Japanese output and prices using the Kalman filter technique. The data used were the quarterly index of industrial production (IIP) and the wholesale price index (WPI) from 1967 to 1989. The author fitted several models with and without a seasonal component. These models were compared to

independent 2-series univariate models and the AIC was used to select the best fitting model. The results showed that fluctuation of WPI had a larger influence on the fluctuation of IIP over that of IIP to WPI. In addition, the model with a seasonal component was found significantly better than the model without seasonal component and the model with two independent univariate series.

Brooks *et al*. (1998) investigated the conditional time-dependent beta series using multivariate GARCH, a market model suggested by Schwert and Seguin (1990) and the Kalman filter technique. The approaches were applied to a sample of returns on Australian industry portfolios from 1974 to 1996. The Kalman filter technique used the first two observations to establish the prior conditions and then recursively estimated the entire series to provide the conditional estimates of dependent beta. Using the measures of forecasting errors, performance of the three approaches were compared. The results provided evidence that the Kalman filter technique outperformed the other models. Also, the improvement of both GARCH and Schwert and Seguin models were quite marked for out-of-sample forecasts.

Franses and Dijk (1996) forecasted stock market volatility using non-linear GARCH models including Quadratic GARCH and GJR. Data used were the weekly observed indices from five European stock markets. The proposed models were then compared to the Random Walk model. Results showed that QGARCH model could significantly improve the symmetric GARCH model and it performed better when the estimation sample excluded the observations during the stock market crisis in 1987. In addition, they concluded that GJR cannot be recommended for forecasting purposes.

Kok and Wong (2003) used Ordinary Least Squared (OLS) and GARCH-M models to examine daily anomalies in Malaysia, Singapore, Thailand, Indonesia and the Philippines during and after the Asian financial crisis. Results showed that when the time-varying volatility was taken into account, there was no significant day-of-the-week effect in the crisis period. In the case of Thailand, the Monday and Friday effects persisted in both the pre-crisis and the post-crisis periods. However, for the other four stock markets, there were no daily anomalies in the post-crisis period.

Arsad *et al*. (2005) used various model from the GARCH family to model the rate of returns of the Kuala Lumpur Composite Index (KLCI) and the Kuala Lumpur Syariah Index (KLSI). The paper also investigated the factors that affected volatility behaviour of the two indices. Daily data of KLCI and KLSI were collected respectively from April 1986 to September 2004 and April 1999 to September 2004. The results showed that the ARMA model with time-varying conditional variances outperformed the constant variance ARMA model. In addition, the results provided evidence of positive risk-return trade-off and significant leverage effect in the Malaysian equity market. Analyses with presence of other economic factors showed the existence of a bi-directional relationship between KLCI and KLSI. Moreover, it was found that returns volatility was affected by trading volume and the inter-bank interest rates.

There are many other econometric approaches used to model the volatility of stock prices. Hardy (2002) used the regime-switching lognormal model (RSLN) to model monthly data from Standard and Poor 500 indices (S&P) and the Toronto Stock Exchange 300 indices (TSE). The performance of the regime-switching model was compared to other common econometric models. The results showed that RSLN-2 provided significant

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improvement over all other models for TSE. However, the results are not very definite for the S&P data. In addition, the RSLN model was applied to maturity guarantees under equity-linked insurance and equations for quantile and derived conditional tail expectation (Tail-VaR) risk measures.

Ibrahim (2003) applied cointegration and VAR modeling to investigate the long-run relationship between the Malaysian equity market, various economic variables and the equity market of US and Japan. The stock indices used were KLCI, S&P 500 and Nikkei 225 Index from January 1977 to August 1998. The results showed the KLCI to be positively related to money supply, consumer price index and industrial production index but negatively linked to the exchange rate. Besides, it was found that the nature of long-run relationships in the Malaysian and Japan equity market were similar but were different from that of the US market. The possible explanation is that Malaysia and Japan are considered as one East Asian market whereas the US market is an alternative market.

The rest of this paper is organised as follows: Section 3 describes the methodology and data used in this paper. Discussion of results is found in Section 4. Section 5 concludes this study.

3. Data and Methodology

Data used in this study was the monthly Kuala Lumpur Composite Index (KLCI) covered from May 1986 to February 2005. The first 200 data were used for the estimation process while the remaining 26 data for forecasting purpose. The data was also divided into two sub-samples in this study: before the Asian crisis (May 1986 to August 1998) and after the Asian crisis in 1997 (September 1998 to February 2005). To obtain the stationary series, the index prices were transformed into returns using the formula as below:

$$R_t = \log(P_t / P_{t-1}) * 100 \tag{1}$$

where R_t is the returns of KLCI, P_t is the stock price at last trading day on month t. All the data series were retrieved from *http://bnm.gov.my*.

Engle (1982) proposed a time-varying conditional variance model which is known as Autoregressive Conditional Heteroscedasticity (ARCH) model. The conditional distribution for a noise series that follows an ARCH(s) process can be written as

$$h_t = \beta_0 + \beta_1 \varepsilon_{t-1}^2 \tag{2}$$

where Ψ_t represents all information available at time t and ε_t is white noise. The variance Equation (2) above is postulated to be a linear function of the past squared innovations.

Bollerslev (1986) developed GARCH(r;s) which would need past variances to describe the future variances. The model is given by

$$h_t = \alpha_0 + \sum_{i=1}^{s} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{r} \beta_j \sigma_{t-j}^2 + \varepsilon_t \tag{3}$$

All the coefficients must be positive so that the model is well defined and the variance is non negative. Equation (3) shows that the present value for conditional variance is a function of a constant, past values of residuals squared and past values of conditional values.

The sum of ($\sum_{i=1}^{s}\alpha_i + \sum_{j=1}^{r}\beta_j$) measures the level of volatility persistency such that the level of volatility persistence increases as the sum approaches unity.

EGARCH was proposed by Nelson (1991) who states that the presence of variance depends on both the size and the sign of the lagged residuals. The model is given as follows:

$$\ln h_t = \lambda_0 + \lambda_1 \left| \frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}} \right| + \lambda_2 \frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}} + \lambda_3 \ln h_{t-1} \tag{4}$$

Log of the conditional variance implies that the conditional variance is non negative, no matter what the sign of the estimated parameters. By using hypothesis that $\lambda_2 < 0$, the presence of leverage effect can be tested.

Engle *et al*. (1987) developed the ARCH-in-Mean (ARCH-M) model by introducing the conditional standard deviation into the mean equation as given by

$$z_t = \mu + \gamma\sqrt{\sigma_t^2} + \varepsilon_t \tag{5}$$

where σ_t^2 follows the GARCH process as in Equation (3). This model is normally used in financial application where the expected return on an asset is related to the expected asset risk. Therefore the coefficient on the expected risk is interpreted as a measure of the risk-return tradeoff.

The Kalman filter technique can be applied to any model that can be written in the state-space form. A state-space model consists of two equations: a measurement equation and a transition equation. The measurement equation describes the relation between the observed variables, z_t and the unobserved state variables. Meanwhile, dynamics of the state variables is described by transition or state equation. A pure Kalman filter is given as

$$z_t = A\theta_t + \varepsilon_t^z \tag{6}$$

$$\theta_t = \Omega\theta_{t-1} + S\varepsilon_t^\theta \tag{7}$$

$$\varepsilon_t^z \sim i.i.d.\mathrm{N}(0,C), \quad \varepsilon_t^\theta \sim i.i.d.\mathrm{N}(0,R), \ \mathrm{E}(\varepsilon_t^z\varepsilon_t^\theta) = 0. \tag{8}$$

where z_t and θ_t are $n\times 1$ vectors of observed variables and $m\times 1$ vector of unobserved state variables at time *t* respectively. A, Ω and S are matrices of parameters of dimension *(n x m)* and *(m x m)* respectively. Refer to Harvey (1989) for more details.

For a Kalman filter-GARCH combination model, the measurement error ε_t^z is subjected to GARCH effect, h_t. Harvey *et al*.(1992) augmented the heteroskedastic shocks into the state vector in the transition equation in the following way:

$$\begin{bmatrix}\theta_{1t}\\ \theta_{2t}\\ \varepsilon_t^z\end{bmatrix}=\begin{bmatrix}\alpha_1 & 1\text{-}\alpha_1 & 0\\ 0 & \alpha_2 & 0\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}\theta_{1t-1}\\ \theta_{2t-1}\\ \varepsilon_{t-1}^z\end{bmatrix}+\begin{bmatrix}S_{11} & 0 & 0\\ 0 & S_{22} & 0\\ 0 & 0 & \sqrt{h_t}\end{bmatrix}\begin{bmatrix}\varepsilon_{1t}^{\theta}\\ \varepsilon_{2t}^{\theta}\\ \varepsilon_t^z\end{bmatrix} \tag{9}$$

The measurement equation will then be written as follows:

$$z_t=\begin{bmatrix}1 & 0 & 1\end{bmatrix}\begin{bmatrix}\theta_{1t}\\ \theta_{2t}\\ \varepsilon_t^z\end{bmatrix} \tag{10}$$

θ_t denotes the series of data based on Wilkie (1986) and Arsad (2002), θ_t is the mean reversion level of the series. Let z_{t-1} be the complete set of observations up to and including time t-1. Given the starting values θ_0 and P_0, the Kalman filter algorithm starts with forecasting a one-step-ahead estimate for the state vector $\hat{\theta}_{t|t-1}$ and its corresponding mean square error (MSE) of the estimate $P_{t|t-1}$. The one-step-ahead estimate and its MSE are given by Equation (11) and Equation (12) respectively:

$$\mathrm{E}(\theta_t \mid Z_{t-1})=\hat{\theta}_{t|t-1}=\Omega\hat{\theta}_{t-1|t-1}=\Omega\theta_{t-1} \tag{11}$$

$$P_{t|t-1}=\mathrm{E}[(\theta_t-\hat{\theta}_{t|t-1})(\theta_t-\hat{\theta}_{t|t-1}))^T]=\Omega P_{t-1|t-1}\Omega^T+R \tag{12}$$

The forecast value of z_t is given by

$$\hat{z}_{t|t-1}=A\mathrm{E}(\theta_t \mid Z_t)=A\hat{\theta}_{t|t-1} \tag{13}$$

with corresponding MSE given by

$$\mathrm{E}[(z_t-\hat{z}_{t|t-1})(z_t-\hat{z}_{t|t-1})^T]=\mathrm{E}[(A(\theta_t-\hat{\theta}_{t|t-1})(\theta_t-\hat{\theta}_{t|t-1})^T A^T]+\mathrm{E}[\varepsilon_t^z(\varepsilon_t^z)^T]$$
$$=AP_{t|t-1}A^T+C \tag{14}$$

Once the new observation z_t is obtained, the one-step-ahead estimate of state variable and its mean square error can be updated. This has to be done to incorporate new information supplied by z_t. Thus the updated estimate $\hat{\theta}_{t|t}$ and its corresponding MSE, $P_{t|t}$ are given by

$$\hat{\theta}_{t|t}=\hat{\theta}_{t|t-1}+P_{t|t-1}A^T(AP_{t|t-1}A^T+C)(z_t-A\hat{\theta}_{t|t-1}) \tag{15}$$

$$P_{t|t}=P_{t|t-1}K_tAP_{t|t-1} \tag{16}$$

where $K_t=P_{t|t-1}A^T(AP_{t|t-1}A^T+C)$ is known as *Kalman gain*.

The likelihood of the state space model is given from the density of the observations $z_1, z_2, ..., z_N$. The density function is decomposed into a product of conditional density functions:

$$L(z;\Psi) = f(z_1 \mid Z_0;\Psi) f(z_2 \mid Z_1;\Psi)......f(z_N \mid Z_{N-1};\Psi) \quad (17)$$

where $\Psi = \{A, C, \Omega, S\}$ is an unknown parameter vector. If the disturbances initial state vector has proper multivariate normal distributions, the distribution of z_t is itself normal:

$$z_t \mid Z_{t-1} \sim \mathrm{N}(A\hat{\theta}_{t|t-1}, F_t) \quad (18)$$

where $F_t = AP_{t|t-1}A^T + C$. Therefore the log likelihood function can be written as

$$\log L \propto -\frac{1}{2}\sum_{t=1}^{N} \log|F_t| - \frac{1}{2}\sum_{t=1}^{N} w_t^T F_t^{-1} w_t \quad (19)$$

where $w_t = z_t - \hat{z}_{t|t-1}$ is the vector of prediction errors. Equation (19) is also known as the *prediction error decomposition form* of the likelihood.

4. Results and Discussion

Table 1 shows the comparison of statistical goodness of fit (within sample) for pure Kalman filter (KF) and various types of KF-ARCH-type models, whereas Table 2 shows the comparison of statistical goodness of fit for pure ARCH-type and KF-ARCH-type models. It can be seen in Table 1 that the KF model has the smallest log likelihood and largest value of AIC among the models. The results highlight the fact that KF models with ARCH structure for the residuals better estimate the KLCI series than the traditional Kalman filter model. From Table 2, note that all KF-ARCH-type models tabulated give higher values of log likelihood than their corresponding ARCH models. For the KF-ARCH-type models, the smallest value of AIC can be found in the KF-GARCH-M model. The results coincide with the highest value of log likelihood. Generally, a comparison among the models strongly supports the use of KF-GARCH models compared to the pure Kalman filter and GARCH models respectively.

Table 1. Comparison of statistical goodness-of-fit for various types of KF-ARCH-type and KF Models

Model	Log L	Rank	AIC	Rank	BIC	rank
Kalman Filter (KF)	-538.24	5	1084.47	5	1097.67	3
KF-ARCH	-533.35	3	1078.70	3	1098.49	4
KF-GARCH	-527.16	2	1068.34	2	1091.42	1
KF-GARCH-M	-525.77	1	1067.54	1	1093.93	2
KF-EGARCH	-531.80	4	1077.60	4	1100.69	5

Table 2. Comparison of statistical goodness of fit for various types of KF-ARCH-type and pure ARCH-type models

Model	Log L	AIC	BIC
ARCH	-541.49	1086.98	1093.58
KF-ARCH	-533.35	1078.70	1098.49
GARCH	-527.68	1061.36	1071.25
KF-GARCH	-527.16	1068.34	1091.42
GARCH-M	-527.24	1062.48	1075.67
KF-GARCH-M	-525.77	1067.54	1093.93
EGARCH	-539.32	1084.64	1094.53
KF-EGARCH	-531.80	1077.60	1110.69

Table 3. Estimated parameters for conditional mean equation for KF and various types of KF-ARCH-type models

Model	α_1	α_2	S_{11}	S_{22}
Kalman Filter (KF)	0.3358	-0.6272	-7.0756	7.1866
	(0.0911)	(0.1285)	(0.9823)	(2.7951)
KF-ARCH	0.1798	0.9910	-6.9937	-0.0000005
	(0.1329)	(0.0032)	(0.3578)	(0.1224)
KF-GARCH	0.5306	0.6942	-2.2521	0.0347
	(0.2227)	(0.1649)	(1.1848)	(1.9374)
KF-GARCH-M	0.5037	-0.8423	3.1056	-3.300
	(0.1531)	(0.1472)	(0.9139)	(1.2899)
KF-EGARCH	0.3684	0.9933	5.2823	0.00003
	(0.1089)	(0.0126)	(0.3914)	(0.1582)

Table 3 shows the estimated parameter for conditional mean equation for KF and various types of KF-ARCH models. The estimates for α_1 and α_2 suggest that (for example, the KF model) each month the KLCI is equal to 33.6 per cent of the last month price plus 66.4 per cent of the last month's deviation of the mean reversion level from the mean, plus a random noise. Besides, it is shown that the contribution from the last month's deviation of the mean reversionary level of KLCI is two times larger than that from last month's KLCI price (-0.6272 and 0.3358 respectively). Note that for all the models, the estimates for α_2 are higher than the estimates for α_1. It is also observed that the estimate of for all the above models is positive. The negative signs of for pure KF and KF-GARCH-M show high uncertainty for the unobserved series. From Table 3, it is found that the estimate of the noise in the stated equation S_{11} and S_{22}, are sensitive to the type of model. Clearly, the KF model has the largest magnitude of noise among the models (7.0756 and 7.1866 respectively). As a result, it provides less satisfactory results for the model. Note that most of the estimates for S_{22} are smaller than S_{11}. Although the opposite result is observed for pure KF and KF-GARCH-M, the difference between S_{11} and S_{22} is small.

Table 4. Estimated parameter for conditional variance equation for pure ARCH-type and various KF-ARCH-type models

Model	β_0	β_1	β_2	γ
ARCH	74.8490 (5.6778)***	0.1094 (0.0969)	- (-)	- (-)
KF-ARCH	1.2832 (0.3823)***	0.9831 (0.0238)***	- (-)	- (-)
GARCH	4.1527 (1.7477)***	0.1777 (0.0673)***	0.7834 (0.0468)***	- (-)
KF-GARCH	3.2010 (0.9044)***	0.2292 (0.0000)***	0.7493 (0.0000)***	- (-)
GARCH-M	4.4702 (2.0267)***	0.1919 (0.0742)***	0.7709 (0.0540)***	0.0810 (0.0760)
KF-GARCH-M	2.9276 (0.1887)***	0.2981 (0.0359)***	0.6710 (0.0372)***	0.0274 (0.0011)***
	λ_0	λ_2	λ_3	
EGARCH	3.8838 (0.0000)***	-0.4362 (0.0000)***	0.0143 (0.0000)***	
KF-EGARCH	0.6598 (0.0852)***	-0.2685 (0.1873)*	0.6754 (0.0187)***	

Note: *** and * denote significance at 1 and 20 per cent levels respectively.

Table 4 shows the estimated parameters for conditional variance equation for GARCH and various types of KF-GARCH models. Most of the parameters are found to be statistically significant at 1 per cent significance level. For KF-GARCH model, the ARCH and GARCH terms (0.2292 and 0.7493 respectively) the sum is less than unity and satisfies the positivity assumptions outlined in Section 3. This finding indicates that shocks to the stock market have a high persistent effect and that high volatility decays at a slow pace. For the KF-GARCH-M model, it is found that the estimated coefficient on the expected risk, γ is statistically significant at 1 per cent level. Thus, the results provide evidence of positive risk-return trade-off in the Malaysian stock market. The positive risk-return trade-off implies that investors will be compensated by higher returns for bearing a higher level of risks, which corresponds with the Capital Asset Pricing Model (CAPM) proposition. From Table 4, note that the estimated parameter of leverage effects, λ_2 from KF-EGARCH model is statistically significant at 20 per cent level. The result proves the existence of very weak leverage effect in the stock market. In addition, the negative parameter of λ_2 suggests that the occurrence of negative returns will increase volatility more than positive returns.

Table 5 shows both the within-sample and out-of-sample (multi-steps) forecast errors for KF and various types of KF-ARCH-type models. Clearly, results show that the KF model gives the poorest forecasts among the models. This is proven by the highest value of within sample forecast error and out-of-sample forecast errors (8.9587 and 9.1668

Table 5. Within-sample and out-of-sample forecast errors for KF and various KF-ARCH-type models

Model	KF	KF-ARCH	KF-GARCH	KF-GARCH-M	KF-EGARCH
$\sqrt{P_{t\|t-1}}$	8.9587	7.0166	2.5866	4.0814	5.5209
$\sqrt{P_{N+\infty\|N}}$	9.1668	7.1097	2.653	4.2337	5.6826

Table 6. Estimated parameters before and after crisis for the best model (KF-GARCH-M)

Period	α_1	α_2	S_{11}	S_{22}	β_0	β_1	β_2	y
Before crisis	0.5676	-0.8579	2.1521	3.2573	3.4194	0.2560	0.7222	0.0508
	(0.0261)	(0.0318)	(0.0079)	(0.0187)	(0.0382)***	(0.0174)***	(0.0155)***	(0.0Q159)***
After crisis	0.8508	-0.7305	0.00009	3.6763	0.000003	0.1240	0.8759	0.1282
	(0.0076)	(0.0603)	(0.0238)	(0.1775)	(0.0000)***	(0.0008)***	(0.0008)***	(0.0498)***

Note: *** denotes significance at 1 per cent level.

respectively). Among the KF models with ARCH-type disturbances term, KF-ARCH has rather high forecast errors (7.0166 and 7.1097). Meanwhile, within sample and out-of-sample forecast errors for KF-GARCH and KF-GARCH-M are found almost four and two times smaller than that found in the traditional KF model respectively. Comparing Tables 3 and 5, it is noted that the higher estimates of the covariance matrix *S*, the higher the forecast error for the models. In conclusion, all Kalman filter-ARCH-type combination models have smaller forecast errors compared to that of the pure KF model. Figure 2 shows within-sample forecast errors for all the proposed models. In addition, one-step-ahead from the Kalman filter-GARCH-M model (the best model in terms of AIC) is shown in Figure 1(a). Obviously, it is much smoother than the observed KLCI series. This is due to the fact that much of the series consisting of noise has been filtered out, to the extent that in some cases it was felt that a lot of information was lost. Figure 1(b) shows mean reversionary level for the KLCI series. It can be seen that the mean reversionary level varies above and below the long-term mean. It decays at a relatively slow rate due to the high magnitude of α_2 (0.8423).

The Kalman filter-GARCH-M model was also tested using two sub-samples: before crisis and after crisis. The results are shown in Table 6. Note that all the coefficients from variance equation show significance at 1 per cent level for both periods. The results indicate a positive risk-return trade-off (0.0508 and 0.1282 respectively) in the Malaysian stock market before and after the economic crisis in 1997. It is also indicated that before 1997, the current stock price depended almost equally on last value of stock price (0.5676) and that of the unobserved series, the mean reversionary level of KLCI. However, it is found that after 1997, the current price of stock is affected approximately 85.08 per cent by its immediate past value and 14.92 per cent of last value of unobserved series.

5. Conclusion

As mentioned earlier, in this study, the nature and behaviour of the volatility of the Kuala Lumpur Composite Index returns using a combination of Kalman filter and various ARCH-type models was studied. The models were then compared to the corresponding pure Kalman filter and pure ARCH-type models, respectively. In general, the results strongly indicate the importance of modeling time varying volatility in the return series. When ARCH structure was taken into account in the conditional variance of the Kalman filter framework, the model was found to provide better results than the pure Kalman filter model. Further analysis also revealed that a combination of Kalman filter and ARCH-type models better fit the KLCI series than the simple ARCH-type models. Generally, Kalman filter model with time-varying conditional variance can better capture the behaviour of return series and successfully model the changing of variances. In addition, it is also shown that there is evidence of positive risk-return trade-off and weak leverage effect in the Malaysian stock market. The last part of this study also investigated the volatility of stock prices for the periods before and after the economic crisis in Asia in 1997. Results show a significant positive risk return trade-off for both periods.

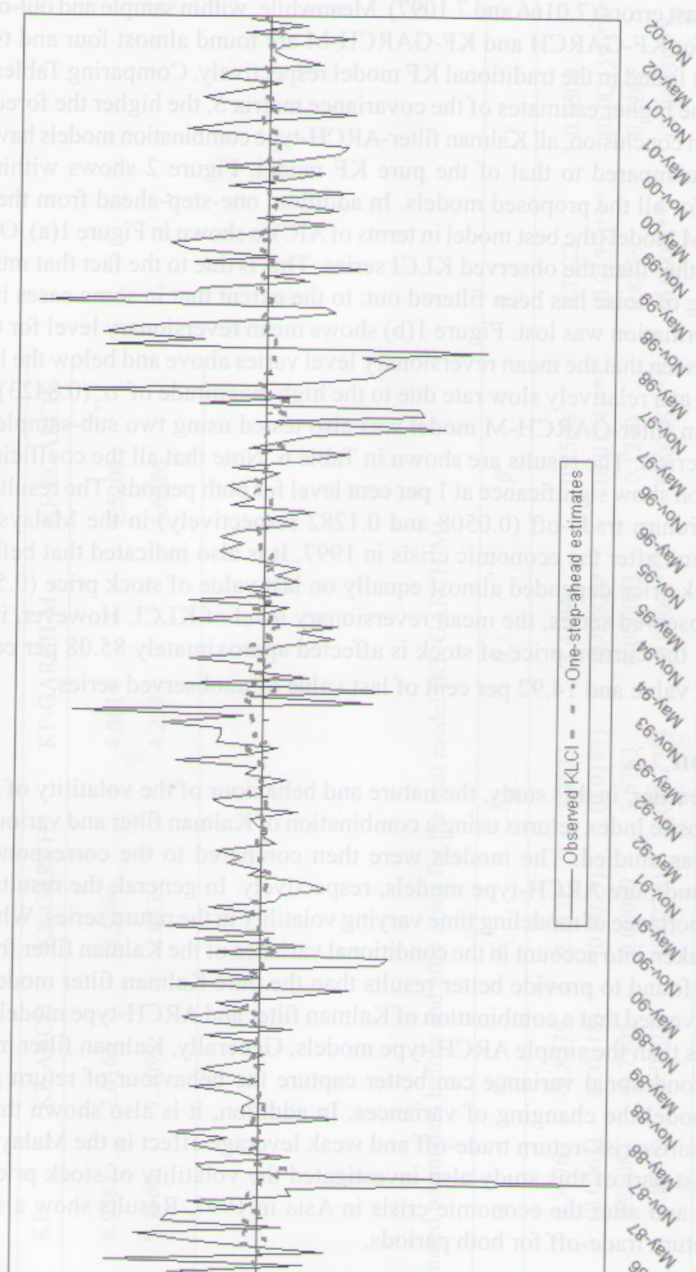

Figure 1(a): KLCI series (dotted line) and one-step-ahead estimates for KF-GARCH-M model

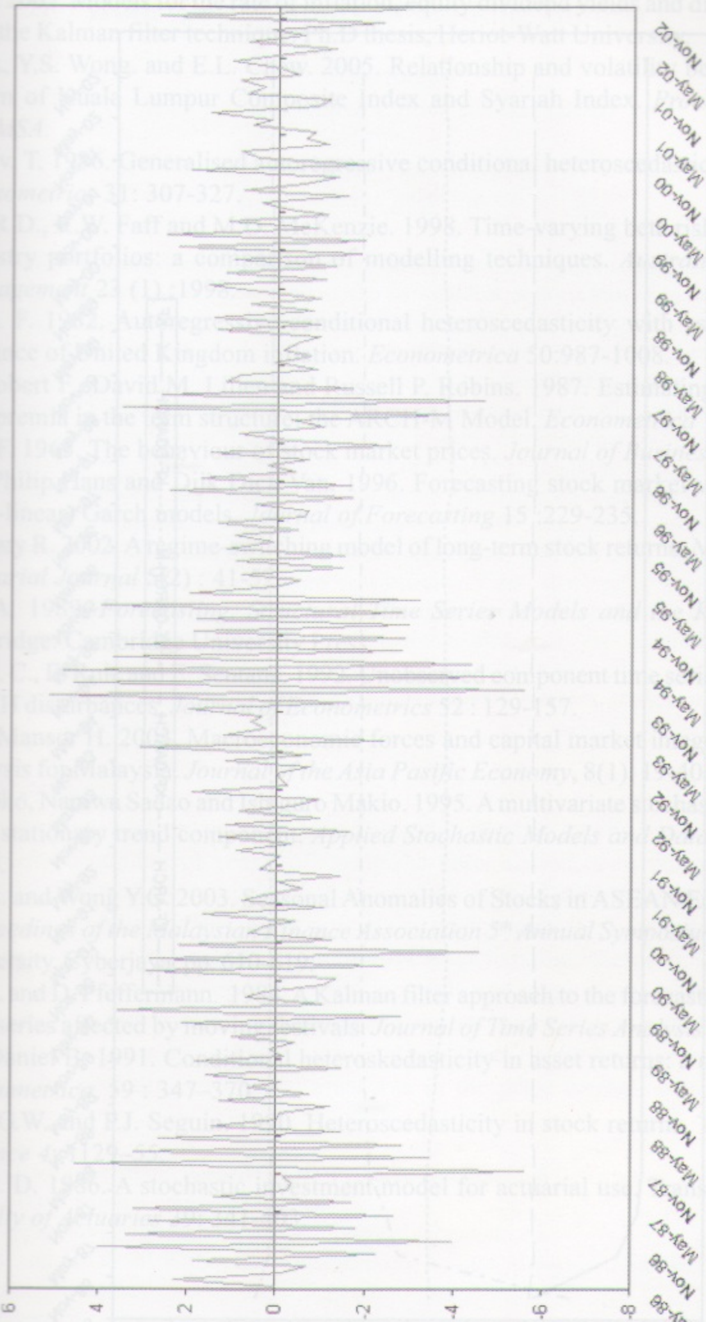

Figure 1(b). The mean reversionary level for KF-GARCH-M model

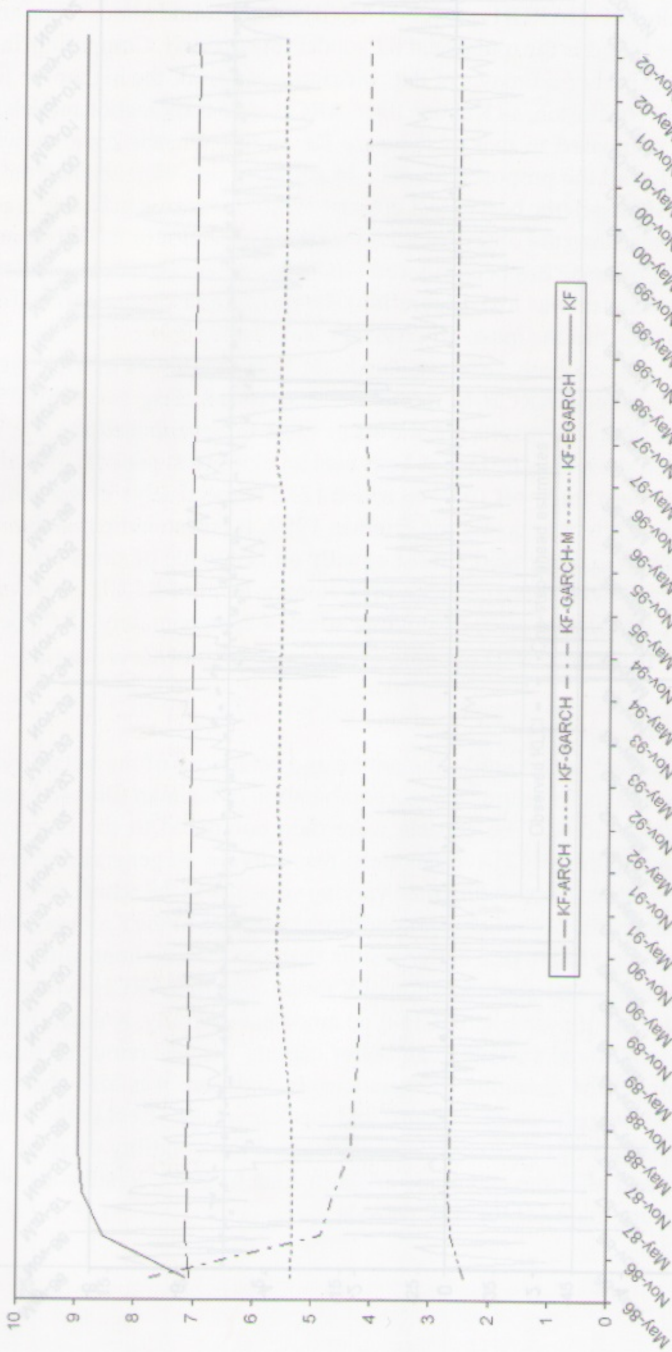

Figure 2. Within sample one-step-ahead estimates forecast errors for KF and various types of KF-ARCH-type models

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