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Weak Form of Call Auction Prices: Simulation Using Monte Carlo Variants

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Abstract: Research Question: This paper explores the pre-market auction price behaviour. The pre-market auction is a short duration auction, where the orders are executed with too little time for revision by the makers. The literature paid attention to application of random walk hypothesis (RWH) and its variants in efficient market (EMH) tests. **Motivation:** The pre-opening auction is an extremely short duration auction where traders are interested in a limited number of large cap stocks and the orders are not transparent. The interest lies on efficiency tests of discrete prices during the pre-market auction for the benefit of investors. **Idea**: The mechanism of price discovery in call auctions is important since they could impact normal markets. We aim to test major relevant hypotheses for pre-opening equilibrium prices. The rejection of the randomness would mean that it is possible to use historical stock prices alone. **Data**: The sample comprises all 50 NSE 50 Index constituent stocks sampled during the year 2019. The NSE constituent stocks maintain the highest market capitalization and have a long history of trading. **Method/Tools**: It summarizes the source literature on objectively driven synthesis on simulation-based decision making since the early period of 1973. Multivariate lognormal distribution is a challenging method than ordinary univariate Monte Carlo. By generating a 50 X 50 covariance matrix of prices and solving for Cholesky roots, the results were compared against lognormal multivariate Monte Carlo simulation to explore the estimates of volatility. **Findings**: The results demonstrate a good case for the tests of RWH and objectively arriving at the pre-opening equilibrium prices. The co-efficient of variation (COV) remained at 3.33%. We found that the stock prices were correlated among themselves, which infers the weak form of efficiency. Previous results had mentioned that MC generated higher sample variances and unsuitable, however, we found lower variances in using multi-variate Monte Carlo. **Contributions**: The contribution lies in the attempts using multi-variate log normal distribution to deduce prices with lower estimated variance. The results have implications to making trade decisions and portfolio construction during the Covid period, where high degree of persisting decline happened to indices.

Keywords: Pre-opening, efficient, multivariate log-normal, Monte Carlo.
JEL Classification: G11, C14, C15, C19

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1. Introduction

An efficient stock market attracts genuine investors to the capital market. The weak form of efficient market hypothesis (EMH) states that a stock's current price would be reflected in the stock's historical prices (Fama, 1991). In a pre-market auction, the individual trade orders are collated and the equilibrium opening prices are derived for each stock and publicly displayed each day. During the short period auction, the makers cannot see the order position of each other, until the final allocation are displayed at the end of the auction. In an efficient market such equilibrium prices may follow Random Walk (RWH). However, due to the very nature of the pre-market, one would expect the equilibrium prices to have some degree of influence with the stock closing prices of the previous trading day (historical). Shiller (1981) identified that stock prices were volatile than their expected discounted cash flows would have made Camerer (1990) observed that prices were volatile and random. Further works on efficient market hypothesis on normal markets had provided mixed inferences. The pre-opening auction prices would follow random walk and could be consistent with the weak form of the EMH (see Malkiel, 2003; Fama, 1991). There exist alternative methods to test for the weak form of market efficiency (e.g., runs test, unit root test, etc.) on normal trading data. However, the technique of Monte Carlo simulation (MC) were not used abundantly in field applications. The MC method is more relevant today because of the uncertainty during the Covid period, where high degree of te persisting decline happened to indices. Technical analysis could not be relied on because of its strong demand on historical prices alone.

There are few questions examined here; how to use MC simulations to stock prices, how accurate is MC method, how to improve the accuracy and reliability for the simulation of the stock prices, respectively? Although, there are many ways of using MC simulation, this study will concentrate on basic approaches with historical data as input parameters; how effectively and efficiently the prediction may result in knowledge to benefit the investors? The results could demonstrate a good case for the tests of RWH and objectively arriving at the pre-opening equilibrium prices. We would find that the stock prices were correlated among themselves, which infers the weak form of efficiency. The contribution would lie in the attempts using multi-variate log normal distribution to deduce prices with lower estimated variance. The results have implications for making trade decisions and portfolio construction during the Covid period.

The rest of the paper is divided into sections; Section 2 highlights the literature review; Section 3 discusses the Hypotheses; Section 4 explains the methodology; Section 5 describes the data; Section 6 describes the results of simulation. Lastly, Section 7 gives concluding remarks.

2. Literature Review

Early discussions of the RWH quote Tversky and Kahneman (1974) who had suggested 'the rule of thumb' in "decision making". The thumb rule occurs when the current price levels were used as "anchors" to arrive at future prices. Later, a growing body of literature paid attention to application of RWH and its variants in testing forms of the EMH. Camerer (1990) had examined price bubbles to distinguish rational response from irrational herd behaviour in the market. Ariely *et al*. (2006) argued that the judgment approach led to biased response. Few studies had devoted to comparing the results between ARIMA (time series) and MC methods in models of stock price, real estate price, interest rate yield, derivative and option prices, etc.

Table 1 summarizes a chronological account of relevant findings in the literature since early period of 1973. It presents relevant source literature in the domain of objectively driven synthesis on simulation based decision making since early period of 1973. As shown in Table 1, many authors have inferred RWH whereas, only a few have negated RWH. Few authors, Chang and Ting (2000), negated RWH by variance ratio tests of the Taiwan Stock Exchange index. Lo and MacKinlay (1998) tested RWH using variance ratio and did not confirm it for indices of for USA. The use of MC for auction prices were few. Specifically, the works of Boyle (1977), Hoesli *et al.* (2005), Whiteside (2008), had been discussed in the literature. Hoesli *et al.* (2005) applied MC simulation to derive and compare the Swiss real estate market prices. Hoesli *et al.* (2005) detected the sensitivity of input parameters of MC. Whiteside (2008) had confirmed that MC could be used to simulate conditional distribution in input functions. The useful properties of MC were highlighted in applications of EMH, and RWH. Cheung and Coutts (2001) demonstrated the weak form of Hang Seng index. Abraham *et al.* (2002) tested RWH for Kuwait, Saudi Arabia and Bahrain markets. Buguk and Brorsen (2003) found some evidence of weak form of the Istanbul exchange. Asiri (2008) confirmed RWH for Bahrain stock prices. Erdos and Ormos (2010) inferred weak-form of the US market. Okpara (2010) inferred random walk in the Nigerian market. Alexeev and Tapon (2011) demonstrated the weak-form efficiency for the Toronto Stock Exchange (TSX).

Table 1: Summary of simulations applications to stock prices

Source	Year	Global	Purpose	Inferences & limitations
Malkiel	1973	TSX	Weak form efficiency for stocks.	Weak form efficiency for stocks.
Tversky and Kahneman	1974	Theory	Psychological bias and heuristic judgments in numeric “decision making”.	Heuristic Judgments could lead to strong bias and systemic errors.
Boyle	1977	USA	MC simulation for option pricing.	MC can be used for numerical forecasting of European stock call options that pay dividends.
Camerer	1990	USA	Asset price bubbles.	Rational response is distinguished from irrational bias observed in the market.
Lo and MacKinlay	1998	USA	RWH random walk using variance ratio.	Do not follow random walks, simple specification test.
Chang and Ting	2000	Taiwan	RWH on variance ratio tests of the stock index.	Negated RWH on variance ratio tests of the stock index.
Cheung and Coutts	2001	Cross Country	Weak-form efficiency.	Confirmed weak form
Abeysekera	2001	India	RWH for the Calcutta Stock Exchange (CSE).	Negated RWH
Abraham *et al.*	2002	Cross Country	Variance ratio test to explore the weak-form.	Confirmed weak form
Buguka and Brorsen	2003	Turkey	RWH for stock prices.	Confirmed RWH for stock prices
Jabbour and Liu	2005	Hang Seng	MC simulation by the number of simulations.	MC simulation is increased by a larger number of attempts in simulations.
Hoesli *et al.*	2005	Theory	Monte Carlo simulation finds its application in much wider areas	MC simulation can be adopted for stocks
Ariely *et al.*	2006	USA	The judgment leads to bias and errors carried over previous prices.	Judgments are unreliable

Table 1 *(continued)*

Source	Year	Global	Purpose	Inferences & limitations
Asiri	2008	Bahrain	Unit root test for RWH for BSE	Prices follow a random walk, confirmed RWH for stock prices
Whiteside	2008	Theoretical	conditional distribution in input functions.	MC could be used to simulate conditional distribution in input functions.
Charles and Darne	2009	Shanghai & Shenzhen	RWH.	RWH was rejected.
Farid *et al.*	2010	Tehran	Generated (VaR) with MC for automobile stocks.	Suggested generating VaR (value at risk) forecast with MC
Erdos and Ormos	2010	USA	Tested weak-form of the US market.	Confirmed weak form
Okpara	2010	Nigerian	RWH	Confirmed random walk
Gupta and Siddiqui	2010	Teheran	RWH and used the Kolmogorov–Smirnov test (K–S) test.	did not exhibit a weak form of market efficiency.
Alexeev and Tapon	2011	Toronto	Demonstrated the weak-form efficiency.	Confirmed weak-form efficiency for the TSE
Landauskas and Valakevičius	2011		Compared standard MC simulation with Markov chain MC simulation (MCMC).	After 300 executed trajectories, the average stock price after 50 trades was exactly the same.
Khan *et al.*	2011	India	Used the unit root test and the GARCH model to test RWH for BSE and NSE	Negated the presence of RWH.
Khan and Vieito	2012	Portuguese	Efficient market hypothesis.	Did not Infer RWH, market was inefficient.
Pant and Bishnoi	2012	India	Used unit root test, autocorrelation and variance ratio to NSE.	RWH was rejected.
Abidin and Jaffar	2014	Malaysian	Implemented MC to Malaysian stocks to find its acceptance.	MC is applicable to stocks
Kyng and Otto	2014	Europe	Multivariate normal Log- normal distribution to European stock options	Suggested multi-asset, multi-period simulation to arrive at the option price.
Mishra *et al.*	2015	India	RWH	Rejected the presence of RWH.
Sonono and Mashele	2015	South Africa	Compared MC with the VG (parametric) model in stock price behaviour in terms of the hit ratios applied to the JSE top 40 index.	GBM worked better than the VG (parametric) model in terms of the hit ratios.
Reddy and Clinton	2016	Australia	Deployed MC for multi-period price samples of large stocks.	Inferred MC as a promising technique for simulation of prices.
Zhang	2020	Asia	Adopted MC for Asian option prices	Derived higher accuracy in a forecast of Asian option prices using Monte Carlo

Notes: Authors' compilation from literature.

However, the tests of RWH by using MC methods specific to call auction price were few. In separate studies, Timothy and Otto (2014), had implemented multivariate distribution model to European stock options to confirm their usability. Abidin and Jaffar (2014) had implemented MC to Malaysian stocks to find its acceptance. Reddy and Clinton (2016) deployed MC to Australian stocks to compare its accuracy over other methods. Boyle (1977) presented the application of the MC to option pricing, who modelled underlying stock returns as continuous and sudden processes. Jabbour and Liu (2005) found that MC simulation accuracy improved with larger number of simulations. Landauskas and Valakevičius (2011) compared standard MC simulation with Markov chain MC simulation (MCMC) on similar trades. After 300 executed trajectories, Landauskas and Valakevičius (2011) found that the average stock price of a sample of 50 trades were identical in both methods. Landauskas and Valakevičius (2011) stated that more number of intervals used in sampling led to higher forecast accuracy. Similarly, the specific tests of accepting RWH were also conducted by few other authors, namely, Charles and Darne (2009) rejected RWH for the Shanghai and Shenzhen markets. Khan and Vieito (2012) inferred that the Portuguese market was inefficient. Sonono and Mashele (2015) showed that the GBM worked better than the VG (parametric) model in stock price behaviour in terms of the hit ratios applied to JSE (South Africa). Zhang (2020) adopted MC for Asian option prices, and, derived higher accuracy in forecast of option prices using MC. Farid *et al*. (2010) suggested generating VaR (value at risk) forecast with MC simulation for automobile stocks in Tehran. Khan *et al*. (2011) used the unit root test and the GARCH model to test RWH for BSE and NSE in India. Gupta and Siddiqui (2010) examined the RWH for the NSE indices in India and used the Kolmogorov–Smirnov test (K–S) test. The results did not exhibit a weak form of market efficiency. Pant and Bishnoi (2012) used unit root test, autocorrelation and variance ratio tests to reject the RWH for NSE. Mishra *et al*. (2015) conduct unit root tests on NSE indices and failed to support RWH. Siddiqui and Patil (2017) also demonstrated that the MC were suited for many Indian stock prices.

3. Hypotheses

As mentioned above, previous findings on RWH tests using methods other than MC based were mixed in nature. Mishra et al (2015) tested RWH on NSE indices and failed to support it. Gupta and Siddiqui (2010) examined the RWH for the NSE indices in India and could not confirm the weak form of market efficiency. We therefore, proceed to test the following hypotheses to characterize the behaviour of pre-opening equilibrium prices.

If the random walk hypothesis is rejected for pre-opening prices, it would imply that historical prices are related to current prices.

H_1: Call auction equilibrium prices follow random walk.

Alternatively, if prices exhibit pure randomness, one cannot foresee prices to his/her benefit.

H_2: The correlation between equilibrium prices today and the previous day are insignificant.

Alternatively, if prices exhibit pure randomness, there is no possibility of correlations among stocks either.

H_3: The correlation between the auction prices of different stocks is insignificant.

If the MC method is an appropriate candidate for simulation, an appropriate distribution exists to achieve accurate forecast prices.

$H_{4:}$ Multivariate lognormal distribution could provide higher accuracy than simple univariate MC.

4. Methodology

The MC (Geometric Brownian Motion) with normal distribution is a case of dimension independence. It differs from numerical analysis whose accuracy could fall with more number of dimensions. The basic notion is that the future prices are conditionally independent of past prices. The common geometric Brownian motion GBM is a Markov process is given as:

$$\Delta P_{i,t} / P_{i,t} = \mu\Delta t + \sigma\varepsilon\sqrt{\Delta t}) \tag{1}$$

where "P" is the stock price, "μ" is the expected return, "σ" (Greek sigma) is the standard deviation of returns, "t" is Time Step, and "ε " (Greek epsilon) is the random variable.

$$\text{or } \Delta P_{i,t} = P_{i,t} (\mu\Delta t + \sigma\varepsilon\sqrt{\Delta t}) \tag{2}$$

where, $\mu\Delta t$ is the drift, and $\sigma\varepsilon\sqrt{\Delta t}$ is the shock. Price drifts up by the expected return for each period. But the drift will be shocked (added or subtracted) by a random term. The stock price follows increments where each increment is a drift plus/minus a random shock of the standard deviation (σ). The residual between the log of Prices (ΔP_t) in two consecutive periods are given as:

$$\varepsilon_t = \text{Log}(P_t) - \text{Log}(P_{t-1}) \tag{3}$$

The residuals "ε_t" represent the prediction error observed to analyze the weak form hypothesis. Subsequently, the Multivariate Monte Carlo Simulation (MVMC) is described by relaxing independence assumption, which takes into consideration the correlation between prices. Suppose that $X = (X_1,..., X_n)$ is a random vector (natural log of equilibrium prices of N stocks), then Σ, the covariance matrix of X, is the (n×n) matrix that has $(i, j)^{th}$ element given by:

$$\Sigma_{i,j} (C)= \text{Cov}(X_i,X_j) \tag{4}$$

The Cholesky Decomposition of Covariance Matrix could reduce the Covariance Matrix to a lower triangular matrix. The Cholesky Matrix is written as:

$$\Sigma = LDL^T \tag{5}$$

where, L is a lower triangular matrix and D is a diagonal matrix with positive diagonal elements. Since the variance-covariance (VACOV) matrix (Σ) is symmetric positive-definite, we can therefore write:

$$\Sigma = LDL^T = (L\sqrt{D})(\sqrt{D}L^T) = (\sqrt{D}L^T)^T(\sqrt{D}L) \tag{6}$$

where, the matrix $C = \sqrt{D}L^T$ satisfies:

$$C^TC= \Sigma \tag{7}$$

where C is the Cholesky Decomposition of Σ. We generate random prices by using a multivariate lognormal random vector with mean μ and variance-covariance (VACOV) matrix Σ', which are passed as input:

$$X = (e^{Y1},...,e^{Yn}) \tag{8}$$

where $Y := (Y_1,...,Y_n) \sim MN(\mu,\Sigma)$. Since, $X = \exp(Y)$, the natural log of prices, Ys are generated. We proceed to implement the above methods to our test sample dataset.

5. Data

The sample comprises NSE 50 Index constituent stocks, which have a long history of trading. The reason for picking NIFTY stocks is the availability of adequate historical data, greater volume of transactions, and consistency as the elements of a popular index. Due to their larger volumes, NIFTY stocks also attract greater institutional interest during the pre-market period. The period of sampling days pertained to the year 2019. The equilibrium prices were collected for consecutive 30 trading days. Each day, the data were collected only after NSE displayed the equilibrium prices after 9.15AM, which is the closing time of pre-opening call.

The descriptive statistics of 50 NSE stocks are given in Table 2, which includes the average equilibrium prices, Std. dev. of prices, average volume of trading, average value of trading, and market capitalization, respectively. The standard deviation of prices varies from 0.82% to 3.53% in the sample. The volume of auctions during pre-market is related to the market capitalization of stocks. There is no observed relationship between standard deviation of prices and market capitalization.

6. Results

The test results against each hypothesis are described here. We proceed with the first 500 simple MC simulation trials. Later, we conduct 500 independent trials for multi-variate log normal distribution. We calculate the t-values and compare the probability of significance for 99.99% confidence limits (95% confidence limit, $t_{TABULATED} = 1.6$, and, 99.99% confidence limit $t_{TABULATED} = 3.46$).

For the correlation tests, we follow Ratner (2009) who suggests that the Pearson coefficient higher than (≥) 0.3 is numerically significant. For the first hypothesis, for example, for the ticker ACC as in Table 2, Average $(\varepsilon_t) = -0.001$; Standard Error $(\varepsilon_t) = 0.00$; $t_{CALCULATED} = | -0.001/0.003| = 0.21$. Since, $t_{CALCULATED} < t_{TABULATED}$, the null hypothesis cannot be rejected. Therefore, auction prices follow a random distribution. We continue to compute the Standard Error (εt) to infer insignificant $t_{CALCULATED}$ for similar other tickers named in Table 1. The results are similar to those reported by Asiri (2008) and Okpara (2010).

We compute the correlation between each stock's residual errors (εt,εt-1) in the second hypothesis separately. As in Table 2, for the first ticker ACC, $\rho_t = -0.31$. Since $\rho_t > 0.3$, it is significant. Therefore, the null hypothesis cannot be rejected. We continue to compute the correlation tests to infer significant $t_{CALCULATED}$ for similar other tickers named in Table 1. The reported tests are similar to the findings of Reddy and Clinton (2016), who have also reported a negative correlation (ρ) during short periods of simulation.

Towards the third hypotheses of correlations between any two stocks, we compute the correlations $(\rho_{i,j})$ between the residual errors in the sample. As in Table 2, for example, between the tickers ACC and Ambuja Cements, we find the correlation $(\rho_{i,j})$ equals 0.67. Since $\rho_{i,j} > 0.3$, it is significant. Therefore, the null hypothesis cannot be rejected.

We continue to compute the correlation tests to infer significant $t_{CALCULATED}$ for similar other tickers named in Table 1. These results are in line with the findings of Schwartz and Whitcomb (1977) and Pant and Bishnoi (2012).

Table 2: Descriptive statistics

No.	Ticker	Average price (INR)	Std. dev. of price (INR)	Average volume (No)	Average value (INR 0.1 mil.)	Average market capitalization (INR 10 mil.)
1	ACC CEMENT LTD	1,462.79	20.43	235.13	3.43	13,677
2	AMBUJA CEMENT LTD	221.18	3.10	2,020.50	4.46	16,906
3	ASIAN PAINTS LTD	572.21	13.51	864.63	4.91	25,796
4	AXIS BANK	1,887.73	37.78	1,019.13	19.14	62,290
5	BAJAJ AUTO LTD	2,217.08	57.53	306.00	6.93	29,615
6	BANK OF BARODA	855.07	16.57	1,575.13	13.43	15,980
7	BHARTI AIRTEL	338.84	2.77	3,413.13	11.57	40,110
8	BHEL ELECTRICAL LTD	245.49	5.58	9,118.63	22.28	22,089
9	BPCL PETROLEUM LTD	572.02	15.81	3,434.75	19.64	14,805
10	CAIRN ENERGY	371.29	4.12	2,078.25	7.73	21,771
11	CIPLA LTD	425.60	4.51	2,075.88	8.85	21,531
12	COAL INDIA LTD	391.07	6.51	5,108.88	19.90	25,518
13	DLF ESTATES	212.71	6.53	10,258.50	21.73	9,410
14	DR.REDDY’S PHARMA	2,455.15	32.36	254.63	6.20	31,135
15	GAIL LTD	448.32	11.11	2,279.13	10.36	20,837
16	GRASIM LTD	3,508.21	53.58	70.50	2.48	22,281
17	HCLTECH	1,426.69	18.19	672.63	9.61	38,473
18	HDFC LTD	979.43	13.94	1,566.13	15.52	153,225
19	HDFC BANK	827.91	8.73	2,206.50	18.19	153,052
20	HEROMOTO CORP	2,606.00	70.36	6,516.38	176.97	31,026
21	HINDALCO	166.69	1.85	5,565.88	9.30	21,305
22	HINDUSTAN UNILEVER	622.26	5.50	717.25	4.48	44,130
23	ICICI BANK LTD	1,425.09	19.47	2,299.75	32.75	163,445
24	IDFC LTD	129.02	1.41	10,658.88	13.70	16,112
25	INDUSIND BANK	563.00	11.42	537.63	3.04	24,407
26	INFOSYS LTD	3,243.67	45.83	1,604.00	52.07	156,655
27	ITC LTD	326.16	10.40	20,984.13	67.04	180,026
28	JINDAL STEEL LTD	326.20	7.16	1,643.25	5.36	11,703
29	KOTAK BANK	911.73	30.28	1,059.75	9.60	34,695
30	L & T LTD	1,675.26	21.81	2,385.13	40.20	135,366
31	LUPIN PHARMA	1,011.56	13.26	326.63	3.30	24,070
32	M &M LTD	1,177.71	25.65	1,661.38	19.60	54,110
33	MARUTI LTD	2,431.41	48.38	776.63	19.01	32,047
34	MCDOWELLS BEVERAGES	2,650.45	93.69	626.88	16.19	20,607
35	NMDC MINERALS LTD	178.96	4.36	2,552.25	4.52	14,091
36	NTPC POWER LTD	154.34	3.33	6,742.13	10.42	31,598
37	ONGC GAS LTD	428.65	12.16	24,910.38	106.29	76,760
38	PNB BANK	967.57	10.85	518.75	5.02	14,352
39	POWERGRID CORP OF INDIA	133.28	1.78	5,026.38	6.69	29,209
40	RELIANCE INDUSTRIES	1,049.36	28.28	10,936.63	113.33	171,092
41	SBI BANK OF INDIA	2,645.71	45.73	1,502.38	39.73	81,454
42	SS SESA STERLITE LIMITED	295.37	2.93	6,065.00	17.93	36,431
43	SUNPHARMA LTD	634.98	7.83	3,859.50	24.94	47,671
44	TATAMOTORS LTD	440.10	4.94	6,225.00	420.28	78,855
45	TATAPOWER LTD	104.21	1.98	5,527.25	5.74	18,782
46	TATASTEEL LTD	529.34	9.19	4,938.38	26.15	35,010
47	TCS SOFTWARE LTD	2,279.74	31.85	655.00	14.94	117,556
48	TECH MAHNIDRA LTD	2,012.53	18.45	1,524.50	30.69	30,042
49	ULTRA CEMENT CO	2,738.54	36.64	95.88	2.64	27,990
50	WIPRO	533.87	4.38	917.00	4.90	34,884

Notes: National Stock Exchange of India (2019)

In continuation, using simple MC, Table 3 produces the summary statistics of simple univariate MC simulation on the sample of 50 stocks.

Table 3: Univariate log-normal MC simulation of 100 trials

No.	Stock Name	Mean	Median	Std dev	Co-efficient of variation	Min	Max
1	ACC CEMENT LTD	1469.75	1467.32	6.75	0.46%	1459.53	1482.76
2	AMBUJA CEMENT LTD	226.91	227.08	0.63	0.28%	225.58	227.84
3	ASIAN PAINTS LTD	551.61	551.32	1.40	0.25%	549.40	555.35
4	AXIS BANK	1879.19	1882.85	13.94	0.74%	1853.48	1898.72
5	BAJAJ AUTO LTD	2133.31	2141.20	17.65	0.83%	1758.70	2608.91
6	BANK OF BARODA	869.60	869.35	3.99	0.46%	861.92	877.28
7	BHARTI AIRTEL	340.83	340.94	1.23	0.36%	337.94	342.92
8	BHEL ELECTRICAL LTD	245.98	246.44	1.39	0.57%	242.51	247.99
9	BPCL PETROLEUM LTD	596.68	595.38	3.65	0.61%	591.25	606.02
10	CAIRN ENERGY	377.08	376.78	1.07	0.28%	375.48	379.29
11	CIPLA LTD	418.36	418.37	1.15	0.27%	414.99	420.24
12	COAL INDIA LTD	394.31	393.60	2.33	0.59%	390.28	398.69
13	DLF ESTATES	229.33	229.33	1.64	0.72%	225.26	233.13
14	DR. REDDY’S PHARMA	2431.52	2432.36	3.59	0.15%	2422.99	2436.96
15	GAIL LTD	438.20	439.35	6.24	1.42%	426.62	447.57
16	GRASIM LTD	3600.88	3604.80	7.66	0.21%	3580.02	3611.85
17	HCLTECH	1401.14	1401.39	3.46	0.25%	1393.34	1408.12
18	HDFC LTD	979.38	978.82	3.36	0.34%	973.17	986.80
19	HDFC BANK	842.44	842.49	0.82	0.10%	840.82	844.79
20	HEROMOTO CORP	2728.85	2726.42	22.19	0.81%	2680.64	2766.53
21	HINDALCO	167.40	167.12	0.96	0.57%	165.80	169.57
22	HINDUSTAN UNILEVER	629.61	629.54	1.52	0.24%	626.79	632.80
23	ICICI BANK LTD	1456.50	1457.12	5.96	0.41%	1445.34	1466.71
24	IDFC LTD	131.77	131.76	0.25	0.19%	131.25	132.37
25	INDUSIND BANK	553.34	553.75	1.84	0.33%	549.20	556.79
26	INFOSYS LTD	3193.93	3193.28	9.51	0.30%	3174.75	3216.85
27	ITC LTD	337.70	337.54	0.92	0.27%	335.66	340.15
28	JINDAL STEEL LTD	342.07	342.95	3.12	0.91%	335.59	348.04
29	KOTAK BANK	929.35	929.24	4.96	0.53%	918.91	938.95
30	L & T LTD	1722.42	1722.49	8.11	0.47%	1700.97	1737.62
31	LUPIN PHARMA	987.87	987.90	2.26	0.23%	983.38	992.32
32	M &M LTD	1230.49	1230.30	3.11	0.25%	1223.93	1238.17
33	MARUTI LTD	2515.63	2511.06	17.68	0.70%	2482.82	2546.98
34	MCDOWELLS BEVERAGES	2779.87	2780.90	29.57	1.06%	2721.64	2844.93
35	NMDC MINERALS LTD	188.46	188.42	1.61	0.85%	185.59	191.60
36	NTPC POWER LTD	161.42	161.32	0.43	0.27%	160.61	162.45
37	ONGC GAS LTD	439.97	441.58	4.60	1.05%	428.53	445.70
38	PNB BANK	979.77	979.87	1.42	0.14%	976.65	982.04
39	POWERGRID CORP OF INDIA	137.93	138.03	0.49	0.36%	137.00	138.85
40	RELIANCE INDUSTRIES	1091.82	1091.48	6.42	0.59%	1081.65	1102.26
41	SBI BANK OF INDIA	2696.16	2697.89	11.29	0.42%	2674.44	2713.64
42	SS SESA STERLITE LIMITED	295.88	295.48	1.55	0.52%	293.25	299.22
43	SUNPHARMA LTD	630.34	630.18	1.35	0.21%	627.65	633.27
44	TATAMOTORS LTD	448.33	448.28	1.14	0.25%	445.83	450.65
45	TATAPOWER LTD	107.21	107.04	0.55	0.51%	106.45	108.35
46	TATASTEEL LTD	543.11	543.21	5.41	1.00%	532.36	550.62
47	TCS SOFTWARE LTD	2246.14	2244.44	6.14	0.27%	2235.29	2260.83
48	TECH MAHNIDRA LTD	1985.22	1985.87	5.14	0.26%	1974.18	1992.67
49	ULTRA CEMENT CO LTD	2775.71	2776.64	7.28	0.26%	2755.85	2787.22
50	WIPRO LTD	530.79	530.96	1.41	0.27%	527.64	533.33

Notes: National Stock Exchange of India (2019)

Table 3 shows the higher deviation between historical volatility and estimates, displaying the nature of a karyolitic curve. As shown in Table 2, the standard deviation of prices varied from 0.82% to 3.53%. However, in simulated output, the standard deviation estimates vary from 0.1% to 1.42%, which have fallen, by about 3/4th of the actual variance. This implies univariate MC simulation cannot be accepted for field use and prediction. Therefore, simple MC generates prices which are not close to the actual.

In continuation, towards the fourth hypothesis Table 4 shows the summary statistics of multivariate MC simulation on the sample of 50 stocks.

The standard deviation varies between 0.68% to 3.33%, which is closer to the historical variation between 0.82% to 3.53%, in descriptive statistics in Table 2. Therefore, we cannot reject the null hypothesis which is the fourth hypothesis. This is in consonance with the findings of Milevsky and Posner (1998). In contrast Clewlow and Strickland (1998), and Hull and White (2012) had mentioned that MC generated higher sample variances and unsuitable, we found multi-variate that reduces the variance.

Overall, the findings are consistent with the weak form of the EMH. The implications of these findings are important to portfolio managers and investors. Since the simulated prices vary within reasonable limit, they can be used as forecast prices. The prediction may result in knowledge to benefit the investors. The novelty of this research stems from the lack of an exact current method that either use semi-parametric or time series techniques to price behaviour. The MC method can potentially become an effective tool to predict and model stock prices. MC simulation has advantages over time series based ARIMA models (Abidin and Jaffar, 2012). Although it is often highlighted that when the number of variables are many in the dataset, PDE (Partial Differential Equations), numerical techniques (or finite difference methods) are less practical. Hence, the Monte-Carlo method provides an effective approach for complex situations such as multi-dimensionality.

7. Conclusions

This paper investigated the mechanism of price discovery in pre-opening call auctions. The pre-opening auction is an extremely short duration auction where traders are interested in a limited number of large-cap stocks. This study examined weak-form efficiency for companies listed in NSE50. By generating a 50 X 50 covariance matrix of prices and solving for Cholesky roots, the results were compared with lognormal multivariate MC simulation to explore the estimates of volatility. We found that the stock prices were correlated among themselves, which infers the weak form of efficiency for pre-market auction prices.

The difference in the results of this study lies in the unique approach of choosing multi-variate log normal distribution to produce a lower estimated variance. The weak-form efficiency is observed using MC evaluation. Monte Carlo method (MC) was not used abundantly in field applications. The prediction may result in knowledge to benefit the investors. This method is more relevant today because of the uncertainty during the Covid period.

There are few limitations in the use of Multivariate lognormal distribution. MC model rely on the assumption of normality of returns. The other limitation is it does not provide comparative analysis against other predictive methods like Martingale. Since the long-run estimates of variances may not remain stable, MC is only suitable for short-run price forecasts. Further works must conduct add more relevant economic factors to improve the performance of MC simulations. Further research may explore innovative variants of Monte Carlo methods to be applied to stock prices, or indices. Lastly, portfolio-level tests of pre-market auction prices, and long-run historical simulation would be beneficial to develop robust trade strategies.

Table 4: Multivariate log-normal MC simulation

No.	Stock Name	Mean	Median	Std. Dev.	Coefficient of variation	Min	Max
1	ACC CEMENT LTD	1468.76	1470.41	20.84	1.42%	1426.00	1515.35
2	AMBUJA CEMENT LTD	224.63	224.61	3.37	1.50%	216.22	231.92
3	ASIAN PAINTS LTD	553.97	553.16	13.64	2.46%	522.06	584.17
4	AXIS BANK	1903.28	1900.67	34.19	1.80%	1827.28	1984.57
5	BAJAJ AUTO LTD	2208.90	2206.94	51.96	2.35%	2059.39	2337.87
6	BANK OFBARODA	871.74	871.61	17.71	2.03%	832.16	914.83
7	BHARTI AIRTEL	342.23	341.90	3.03	0.89%	336.82	350.96
8	BHEL ELECTRICAL LTD	245.85	245.69	4.88	1.98%	235.72	257.47
9	BPCL PETROLEUM LTD	595.58	593.84	15.89	2.67%	562.97	630.90
10	CAIRN ENERGY	374.43	374.94	4.31	1.15%	364.30	383.93
11	CIPLA LTD	415.75	415.98	4.49	1.08%	401.73	423.01
12	COAL INDIA LTD	397.21	396.98	7.02	1.77%	381.21	414.95
13	DLF ESTATES	222.22	222.30	6.61	2.97%	205.61	238.33
14	DR.REDDY'S PHARMA	2441.69	2440.68	35.85	1.47%	2359.94	2520.39
15	GAIL LTD	435.56	436.33	10.34	2.37%	415.55	461.23
16	GRASIM LTD	3558.14	3554.09	53.48	1.50%	3430.47	3685.17
17	HCLTECH	1412.63	1415.42	16.68	1.18%	1368.06	1443.11
18	HDFC LTD	979.86	980.37	14.86	1.52%	928.37	1023.40
19	HDFC BANK	839.46	838.86	9.00	1.07%	819.48	861.26
20	HEROMOTO CORP	2707.60	2704.30	68.66	2.54%	2563.34	2874.28
21	HINDALCO	168.54	168.73	1.93	1.15%	163.61	173.08
22	HINDUSTAN UNILEVER	628.82	628.59	6.03	0.96%	614.78	644.07
23	ICICI BANK LTD	1452.15	1455.39	19.40	1.34%	1402.28	1498.70
24	IDFC LTD	130.81	130.82	1.40	1.07%	127.55	133.99
25	INDUSIND BANK	558.32	557.64	11.51	2.06%	534.11	593.52
26	INFOSYS LTD	3193.89	3196.99	41.74	1.31%	3097.05	3281.43
27	ITC LTD	334.08	334.43	10.67	3.19%	305.80	364.26
28	JINDAL STEEL LTD	334.95	334.91	6.87	2.05%	321.03	350.38
29	KOTAK BANK	926.97	930.32	30.90	3.33%	846.97	1019.43
30	L & T LTD	1719.53	1720.30	19.79	1.15%	1676.22	1766.13
31	LUPIN PHARMA	995.99	994.61	10.78	1.08%	973.10	1022.31
32	M &M LTD	1225.90	1224.77	27.78	2.27%	1149.53	1307.68
33	MARUTI LTD	2480.85	2483.25	46.07	1.86%	2379.03	2575.97
34	MCDOWELLS BEVERAGES	2732.75	2728.40	87.37	3.20%	2509.99	2953.02
35	NMDC MINERALS LTD	185.46	185.56	4.19	2.26%	174.86	195.43
36	NTPC POWER LTD	159.61	160.18	3.20	2.00%	151.78	166.50
37	ONGC GAS LTD	430.00	430.22	9.69	2.25%	408.06	455.38
38	PNB BANK	978.53	978.61	12.05	1.23%	950.15	1010.41
39	POWERGRID CORP OF INDIA	136.48	136.43	1.61	1.18%	132.66	140.03
40	RELIANCE INDUSTRIES	1088.85	1086.04	28.34	2.60%	1027.22	1178.81
41	SBI BANK OF INDIA	2699.76	2698.52	45.65	1.69%	2571.39	2843.33
42	SESA STERLITE LIMITED	293.04	292.82	2.53	0.86%	287.69	301.28
43	SUNPHARMA LTD	634.31	634.05	6.77	1.07%	612.28	648.44
44	TATAMOTORS LTD	447.83	448.35	4.64	1.04%	435.08	456.17
45	TATAPOWER LTD	107.53	107.59	2.14	1.99%	101.48	112.53
46	TATASTEEL LTD	545.38	545.88	8.95	1.64%	522.73	565.68
47	TCS SOFTWARE LTD	2246.90	2248.49	28.59	1.27%	2167.87	2316.08
48	TECH MAHNIDRA LTD	1999.87	1999.21	14.58	0.73%	1969.43	2046.53
49	ULTRA CEMENT CO LTD	2752.81	2756.75	36.35	1.32%	2654.09	2855.53
50	WIPRO LTD	532.82	532.85	3.63	0.68%	524.64	544.58

Notes: National Stock Exchange of India (2019)

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